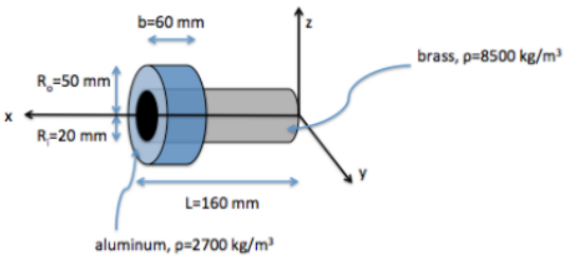
HW8 Solutions, engn0040 2014 (j. franck)

1.



$$I_x = I_{x1} + I_{x2}$$

$$I_{x1} = \tfrac{1}{2}\rho_1 \pi L R_i^4 \quad \text{(cylinder)}$$

$$I_{x2} = \tfrac{1}{2}\rho_2 \pi b (R_o^4 - R_i^4) \quad \text{(annulus)}$$

$$I_x = .00189 \text{ kgm}^2$$

$$I_y = I_{y1} + I_{y2}$$

$$m_1 = \rho_1 \forall_1 = \rho_1 \pi R_1^2 L_1 \qquad m_2 = \rho_2 \forall_2 = \rho_2 \pi b (R_o^2 - R_i^2)$$

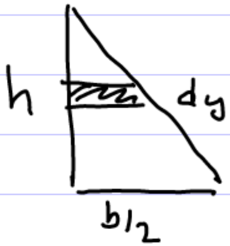
$$I_{y1} = I_{y1G} + \frac{m_1 L_1^2}{4} = \frac{1}{12} m_1 (3R_1^2 + L_1^2) + \frac{m_1 L_1^2}{4}$$

$$I_{y1} = m_1\left(\frac{3R_1^2}{12} + \frac{L_1^2}{12} + \frac{L_1^2}{4}\right) = m_1\left(\frac{R_1^2}{4} + \frac{L_1^2}{3}\right) = .0147 \text{ kgm}^2$$

$$I_{y2} = I_{y2G} + m_2 (L - b/2)^2 = \frac{1}{12} m_2 \left[3(R_o^2 + R_i^2) + b^2\right] + m_2 (L - b/2)^2 = .0192 \text{ kgm}^2$$

$$I_y = .0339 \text{ kgm}^2$$

2. find I_x of half triangle
then mult. by 2



$dA = x\,dy$

$dA = \frac{b}{2}(1 - y/h)\,dy$

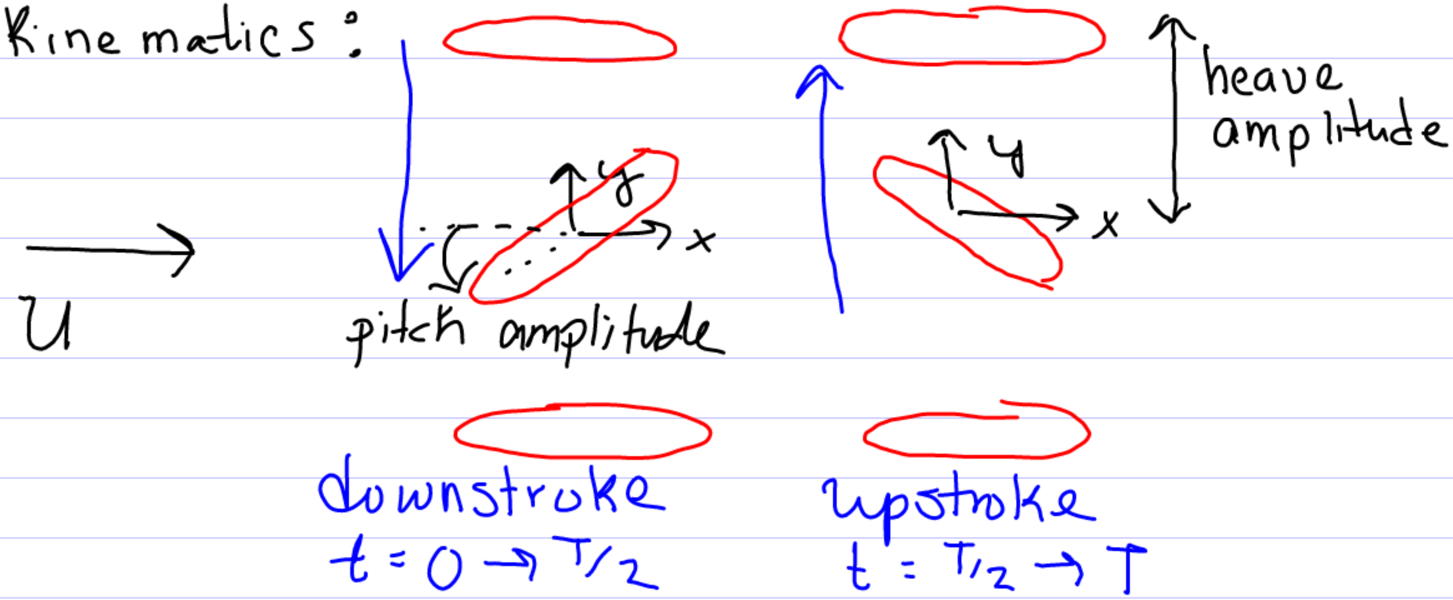
$$I_x = \rho\int y^2\,dA = \rho\int_0^h y^2 \cdot \frac{b}{2}\cdot(1-y/h)\,dy = \rho\frac{b}{2}\int_0^h (y^2 - y^3/h)\,dy$$

$$I_x = \rho b/2\left[\frac{y^3}{3} - \frac{y^4}{4h}\right]\Big|_0^h = \frac{\rho b}{2}\left[\frac{h^3}{3} - \frac{h^3}{4}\right] = \frac{\rho b}{2}\left[\frac{h^3}{12}\right] = \frac{\rho b h^3}{24} \text{ (half)}$$

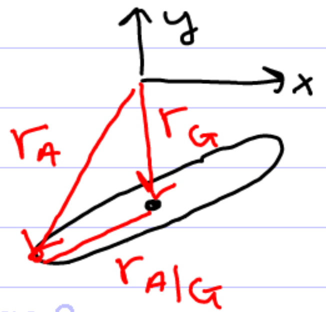
$$h = \sqrt{3}/2\, b \Rightarrow \quad I_x = \frac{\rho b h^3}{12} = \frac{\rho b^4 \cdot 3\sqrt{3}}{12\cdot 8} = \boxed{\frac{\rho b^4\sqrt{3}}{32}}$$

3. Tidal Energy Harvester (See matlab soln too)

Kinematics:



position of A: $r_G = \frac{V_{max}}{\omega}\cos(\omega t)\hat{j}$



$$r_{A/G} = -\frac{c}{2}\cos\theta\,\hat{i} - \frac{c}{2}\sin\theta\,\hat{j}$$

$$r_A = r_G + r_{A/G}$$

$$\boxed{r_A = \left(-\frac{c}{2}\cos\theta\right)\hat{i} + \left(\frac{V_{max}}{\omega}\cos\omega t - \frac{c}{2}\sin\theta\right)\hat{j}}$$

$$\boxed{\text{where } \theta(t) = -\frac{\Omega_{max}}{\omega}\cos(\omega t + \phi)}$$

Kinematics eqns:

$$V_y = -V_{max}\sin(\omega t)$$

$$y(t) = \frac{V_{max}}{\omega}\cos(\omega t)$$

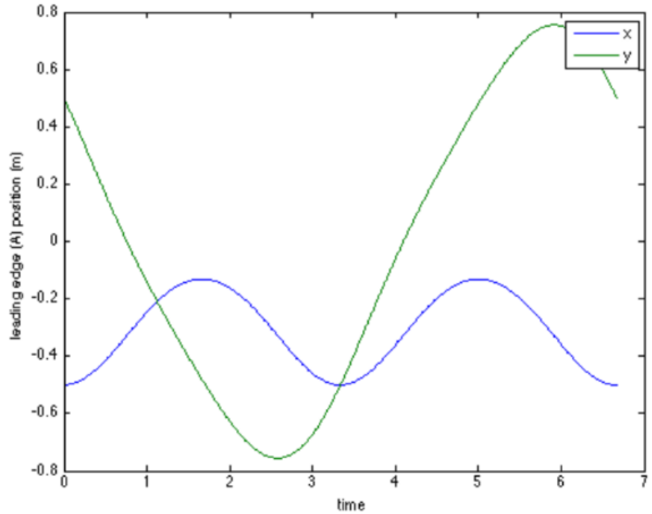
$$y(0) = V_{max}/\omega$$

$$\Omega = \Omega_{max}\sin(\omega t + \phi)$$

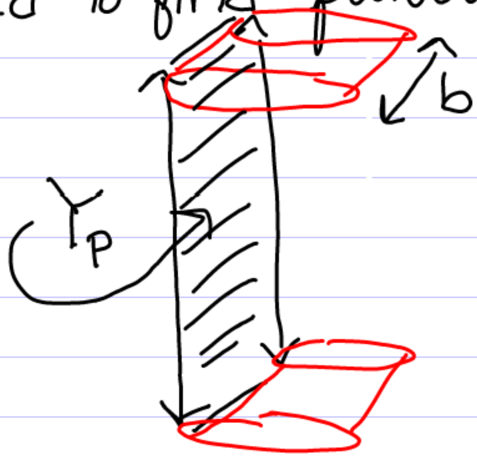
$$\theta(t) = -\frac{\Omega_{max}}{\omega}\cos(\omega t + \phi)$$

$$\theta(0) = 0$$

plotting in matlab:



2. Need to find frontal swept area: $Y_p = (y_{max} - y_{min})b$



can use plot from above

$y_{max} = .7552$ m

$y_{min} = -.7552$ m

$\boxed{Y_p = 15.1 \text{ m}^2}$

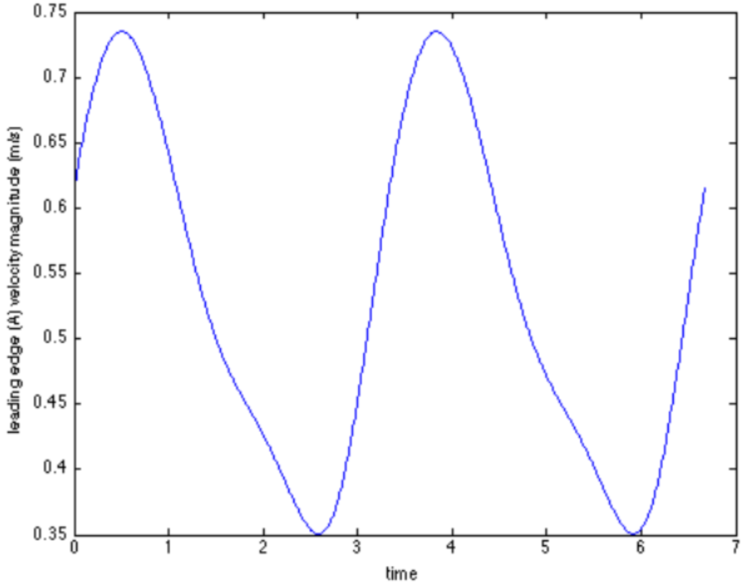
3. velocity will reach max at leading (or trailing) edge, point A

$$\underline{V}_A = \underline{V}_G + (\underline{\Omega} \times \underline{r}_{A/G}) \quad , \quad \underline{V}_G = V_{max} \sin(\omega t)\hat{j} \, , \, \underline{\Omega} = \Omega_{max} \sin(\omega t + \phi)\hat{k}$$

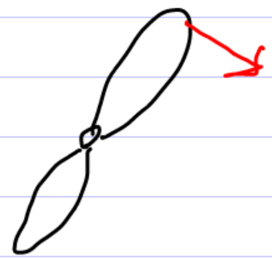
$$\underline{r}_{A/G} = -\tfrac{c}{2}\cos\theta\,\hat{i} - \tfrac{c}{2}\sin\theta\,\hat{j}, \quad \theta = -\frac{\Omega_{max}}{\omega}\cos(\omega t + \phi)$$

$$\underline{V}_A = \underline{V}_G - r_{AGy}\Omega\,\hat{i} + r_{AGx}\Omega\,\hat{j} \Longrightarrow \text{matlab}$$

max from matlab $|V|_{max} = .735\ m/s$



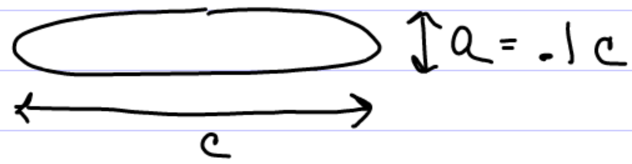
compare to rotary turbine tip speed:



$$V = \omega R = 40\,\frac{2\pi}{60}(2.5) = 10.5\ m/s$$

4. $\alpha(t) = \Omega_{max}\,\omega\cos(\omega t + \phi)$ $\qquad m = \rho V = \rho\pi acb = \rho\pi$

$\alpha_{max} = \Omega_{max}\,\omega$



$M = I_G\,\alpha$ = torque required

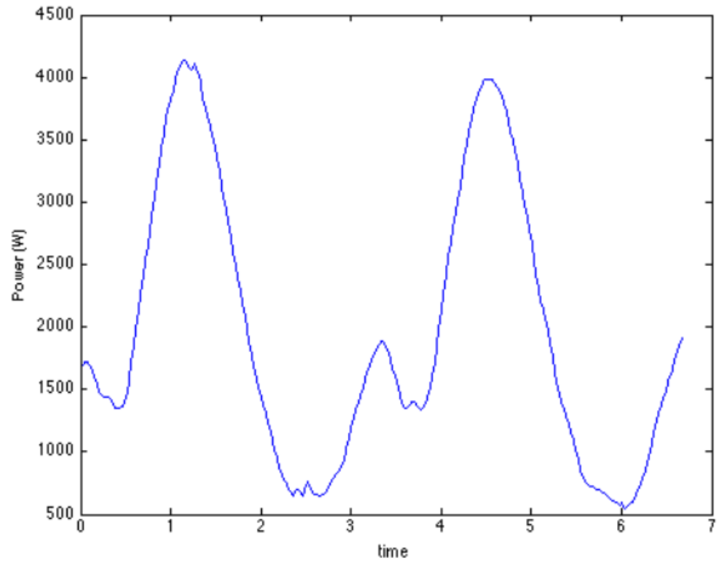
I_G elliptical cylinder $= 1/4\, m(a^2 + c^2) = 1/4\,\rho\pi(1.01) = 119\ kg\,m^2$

$\boxed{M = 53\ N \cdot m}$ in absence of fluid forces

5. $P = v_y F_y + \Omega T_z \Rightarrow$ plot

be sure to convert t into seconds

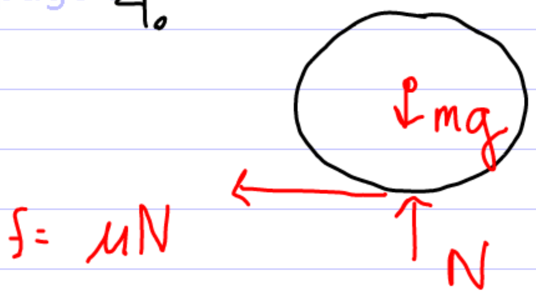
(see matlab)



6. $\bar{P} = 1954\ W$

$\eta = 1954 / \tfrac{1}{2}\rho U^3 Y_p = \boxed{26\%}$

4.



4.2. $\Sigma F_x = -f = -\mu N = ma_x$

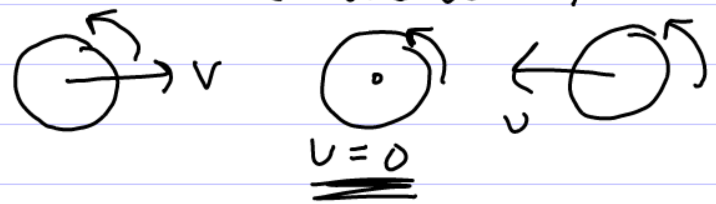
$\Sigma F_y = mg - N = 0$; $N = mg$

$\Sigma M = -\mu N R = I_o \alpha = \frac{1}{2} m R^2 \alpha$

$a_x = -\frac{\mu N}{m} = -\frac{\mu m g}{m} \boxed{= -\mu g}$

$\boxed{\alpha = -2\mu g / R}$

4.3. reverse direction when



Integrate $a_x \Rightarrow v_x = v_0 - \mu g t$

$v_x = 0$ when $\boxed{t = v_0 / \mu g}$

4.4. Roll without slip is when

$v = -\omega R$

Integrate α: $\omega = \omega_0 - \frac{2\mu g t}{R}$

$$v_0 - \mu g t = -\left(\omega_0 - \frac{2\mu}{R} g t\right) R = -\omega_0 R + 2\mu g t$$

solve for t:

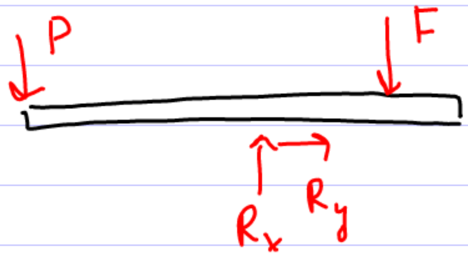
$$\boxed{t = (v_0 + \omega_0 R) / 3\mu g}$$

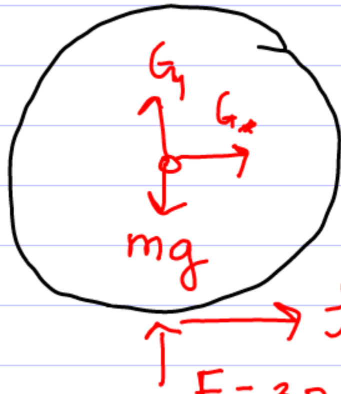
Reversal occurs if $t_{slip} > t_{rev}$

$$(v_0 + \omega_0 R) / 3\mu g > v_0 / \mu g$$

$$\boxed{v_0 < \frac{\omega_0 R}{2}}$$

5.





balance moments about R:

$$-3P + F = 0 \Rightarrow F = 3P$$

Work & Energy: $T_i + U = T_f$ where U = work done by non-con. forces (friction!)

$$T_i = \tfrac{1}{2} I_G \omega^2 = \tfrac{1}{2} \cdot \tfrac{1}{2} m R^2 \omega^2 = \frac{mR^2\omega}{4} = 442$$

$$T_f = 0$$

dist. is arc length

$$U = -f \cdot s = -f R \theta$$

$$T_i - \mu 3 P R \theta = 0$$

Solve $\boxed{P = 7.85\text{ N}}$

where θ is # radians completed in 10 seconds

$$\theta(t) = \omega t + \frac{1}{2}\alpha t^2$$

$\theta = 65.45$ rad in $t = 10$ seconds

Alternative Method

$\Sigma M = fR = I_G\alpha$

$\omega_f = 0 = \omega + \alpha t$, where $\omega = -125 \cdot \frac{2\pi}{60} \frac{rad}{s}$

solve for $\alpha = +1.31\ rad/s^2$

$3P\mu R = \frac{1}{2} mR^2 \cdot \alpha$

$P(3\mu R) = \frac{1}{2} mR^2\alpha$

$\boxed{P = 7.85\ N}$